HYDRAULIC RESISTANCE IN THIN-LAYER SPHERICAL PACKINGS

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Packings composed of spherical items are widely used in compact highly stressed heat and mass exchangers. The performance is dependent to a considerable extent on the hydraulic resistance, which is governed by the structure. The resistance in random (disordered) packings has been examined many times (see [1] for a detailed review). Some papers deal with the resistance in regular (ordered) sphere packings [2]. So far as one can judge from the literature, there is no method of calculating the resistance in a bed of spheres that incorporates the effects from the numbers of rows of spheres, the channel narrowing, and the presence of perforated plates that retain the stack.

Sphere-stack structures have been analyzed in [3], the major geometrical characteristics being the distance between adjacent rows referred to the diameter h/d, the mean-volume porosity ε, which is the ratio of the volume free from spheres to the volume of the channel in which the spheres are contained, and the relative minimal through cross section (clear space) ψ, which is ratio of the area free from spheres to the area of the channel section. Those parameters are [3] related by

$$\frac{h}{d}=\frac{2}{3}\frac{1-\psi}{1-\varepsilon}.$$

As regards external hydrodynamics, the characteristic velocity is taken as the mean flow-rate velocity of the liquid in the channel ahead of the packing w, while the characteristic linear dimension is the sphere diameter d. The hydraulic-resistance coefficient for a sphere in the packing is [4]

$$\xi=\frac{2}{3}\frac{\varepsilon\psi^2}{1-\varepsilon}\zeta\frac{d}{H},$$

in which $\zeta = 2\Delta P/\rho w^2$ and $H = (z - 1)h + d$ is the height of the stack, h the distance between adjacent rows, z the number of rows in the stack, ΔP the pressure difference, and ρ the density of the working medium.

We have made measurements on the resistance in thin-layer spherical arrays for the range in Reynolds number $Re = wd/\nu = 5\cdot10^3$-$2\cdot10^4$ (ν is kinematic viscosity) with an open-type wind tunnel as described in [2, 5]. The air pressure was ~0.1 MPa and the temperature ~300 K. We measured the differences in static pressure and temperature in the working and flowmeter parts by means of U manometers or MMN-240 micromanometers together with laboratory thermometers with scale divisions of 0.5 K.

In this Re range, the resistance for a row of spheres in a stack is determined by the extent to which the flow expands behind the row [2], so the hydraulic resistance is the same in a channel of constant cross section for all the rows of spheres apart from the last, and the pressure difference across a z-row stack is

$$\Delta P=\left[\frac{3}{2}\frac{1-\varepsilon}{\varepsilon\psi^2}\left(\frac{H}{d}-1\right)\xi+\zeta'\right]\frac{\rho w^2}{2}$$

in which ζ' is the resistance coefficient for the last row, which is equal to that for a single row of spheres.

If the spheres are located in a tapering channel, the characteristic flow speed around a sphere in row i is $w_i = Q/F_i$, in which Q is the volume flow rate and F_i the channel area in the middle section of row i. If we take the taper as uniform, the continuity equation gives

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TABLE 1

No.	Packing type [3]	Channel section, mm	ψ	h/d	ξ_0
1	Cubic	113×68	0,224	0,1	0,23
2	Rhombic			0,866	0,30
3	Orthorhombic	120×68	0,123	1,0	0,30
4	Birhombic			0,866	0,25
5	Tetrahedral	113×61	0,135	0,816	0,23
6	Octahedral	113×68	0,224	0,707	0,16

TABLE 2

No.	Stack type [3]	z	σ	$F_z\psi/F_0$	$F_z\psi/F$	ψ	ε
1	Cubic	2 3 4 5	1,0 2,28 3,82 5,64	0,185 0,165 0,149 0,136	0,244	0,220	0,480
		3 5	2,81 7,46	0,110 0,088	0,268	0,190	0,460
2	Rhombic	2 3 4 5	1,0 2,25 3,71 5,46	0,189 0,169 0,161 0,142	0,244	0,222	0,401
		3 5	2,39 6,67	0,129 0,098	0,231	0,190	0,376
		3 5	2,47 7,25	0,073 0,110	0,220	0,210	0,390
		3 5	2,16 5,09	0,175 0,148	0,226	0,220	0,400
3	Birhombic	2 3 4 5	1,0 2,31 3,94 5,90	0,112 0,101 0,090 0,081	0,161	0,120	0,323

$$\Delta P = \left[\frac{3}{2}\frac{1-\varepsilon}{\varepsilon\psi^2}\frac{h}{d}\sigma\xi + \zeta'\right]\frac{\rho w_0^2}{2}$$

$$\left(\sigma = \sum_{i=1}^{z-1}\left[1-(1-F_z/F_0)\frac{i-1}{z-1}\right]^{-2}\right).$$

We examined stacks of spheres with various structures in straight and tapering channels with the number of sphere rows ranging from one to six. Tables 1 and 2 give the geometrical characteristics for those channels correspondingly. For Re > $(4\text{-}6)\cdot10^4$, the resistance coefficient becomes self-similar with respect to Re: $\xi = \xi_0$ and $\zeta' = \zeta_0'$. Our measurements fit satisfactorily to the following formulas (Figs. 1 and 2 correspond to Tables 1 and 2):

$$\xi/\xi_0 = \zeta'/\zeta_0' = 7/\sqrt{\mathrm{Re}} + 1,$$

in which the resistance coefficient for the last row with inertial flow is defined by the Bord–Carnot formula $\zeta_0' = (F_0/(\psi F_z) - F_0/F)^2$ (F_0, F_z, and F are the areas of the channel

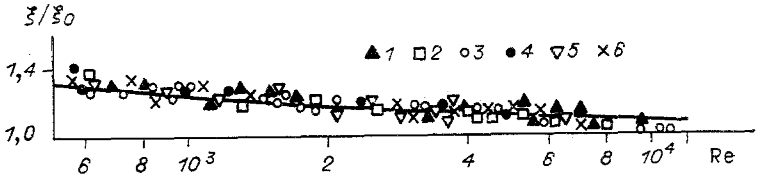

Fig. 1

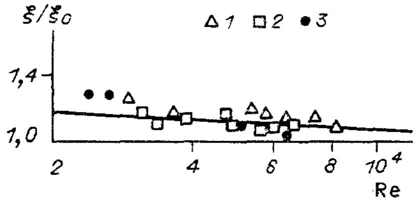

Fig. 2

ahead of the sphere stack, in the middle section of the last row, and behind the stack correspondingly), while ξ_0 is dependent on the structure; values derived from our experiments are given in Table 1. For random sphere packing [1], $\xi_0 = 1.25\varepsilon^{1.5}$ (applicable for ε of 0.26-0.48).

We also examined the hydraulic resistance in packages composed each of sphere stacks placed between perforated plates [6]. The plate porosities were m = 0.05-1.0. The total resistance of a package is defined by

$$\zeta_\Sigma = \sum_{i=1}^{3} \zeta_i,$$

in which $\zeta_i = 2\Delta P_i/\rho w^2$ is the coefficient for local resistance i (i = 1 and 2 represent perforated plates and i = 3 the sphere stack).

We did not find any interaction between the local resistances in these packages, although there is substantial interaction between paired perforated plates free from sphere packings, which can be incorporated from the formula [6]

$$\zeta_\Sigma = (1 + K_i) \sum_{i=1}^{2} \zeta_i$$

in which $K_i = 0.18(\ell/D) - 0.5$ is the interference coefficient, ℓ/D = 0-14, ℓ the gap between the plates, and D the hydraulic channel diameter.

Formulas have been derived that reflect how the resistance coefficient is affected in a thin-layer sphere stack by the stack structure, number of sphere rows, channel taper, and perforated plates that retain the spheres.

LITERATURE CITED

1. N. N. Suntsov, S. L. Demenok, and A. I. Gusev, Fitting Formulas to Measurement on the Hydraulic Resistance of Sphere Packings [in Russian], Dep. VINITI No. 2806-V89 (1989).
2. N. N. Suntsov, S. L. Demenok, and V. V. Medvedev, Hydraulic Resistance in Regular Sphere Packing [in Russian], Dep. VINITI No. 8766-V88 (1988).
3. S. L. Demenok, D. V. Lavrenov, and N. N. Suntsov, Structures and Geometrical Characteristics of Sphere Packings [in Russian], Dep. VINITI No. 5190-V88 (1988).
4. M. A. Gol'dshtik, Transport Processes in Granular Beds [in Russian], IT SO AN SSSR, Novosibirsk (1984).
5. S. L. Demenok, V. V. Medvedev, S. M. Sivukha, and B. V. Sudarev, Hydraulic Resistance in Thin-Layer Stacks with Regular Sphere Arrays [in Russian], Dep. TsNII Rumb, No. DR-3027 (1988). Published in Sudostroenie, Ser. 2, No. 8 (1988).
6. N. N. Suntsov, B. V. Sudarev, S. M. Sivukha, et al., Hydraulic Resistance in Regular Sphere Stackings Bounded by Perforated Plates [in Russian], Dep. TsNII Rumb, No. DR-2811 (1987). Published in Sudostroenie, No. 8 (1987).